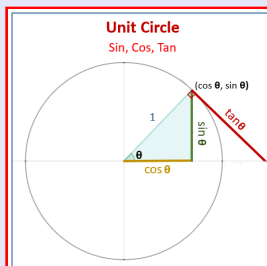


Trigonometry

Lecture 24



Feb 19-8:47 AM

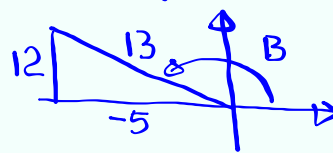
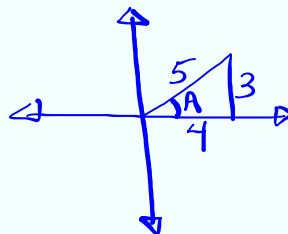
Class Quiz 7

$$\sin A = \frac{3}{5}, \text{ A is in QI}$$

$$\cos B = \frac{-5}{13}, \text{ B is in QII}$$

Find 1) $\sin 2A$

$$\begin{aligned} \sin 2A &= 2 \sin A \cos A \\ &= 2 \cdot \frac{3}{5} \cdot \frac{4}{5} = \boxed{\frac{24}{25}} \end{aligned}$$

2) $\cos(A-B)$

$$= \cos A \cos B + \sin A \sin B$$

$$= \frac{4}{5} \cdot \frac{-5}{13} + \frac{3}{5} \cdot \frac{12}{13}$$

$$= \frac{-20}{65} + \frac{36}{65} = \boxed{\frac{16}{65}}$$

Oct 9-10:25 AM

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$\cos 2A = 2 \cos^2 A - 1 \Rightarrow \cos^2 A = \frac{1 + \cos 2A}{2}$$

$$\cos 2A = 1 - 2 \sin^2 A \Rightarrow \sin^2 A = \frac{1 - \cos 2A}{2}$$

$$\sin A = \pm \sqrt{\frac{1 - \cos 2A}{2}}, \quad \cos A = \pm \sqrt{\frac{1 + \cos 2A}{2}}$$

we determine + or - based on A

Replace A with $\frac{A}{2}$

$$\sin \frac{A}{2} = \pm \sqrt{\frac{1 - \cos A}{2}}, \quad \cos \frac{A}{2} = \pm \sqrt{\frac{1 + \cos A}{2}}$$

Determine + or - based on $\frac{A}{2}$

Oct 9-10:49 AM

$$\sin A = -\frac{3}{5} \quad A \text{ is in QIII}$$

$$\text{Find } \sin \frac{A}{2} \text{ \& } \cos \frac{A}{2}$$

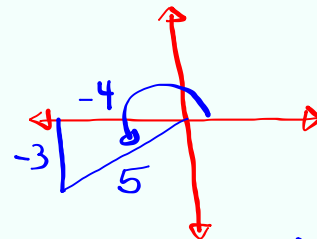
Formula

$$\sin \frac{A}{2} = \pm \sqrt{\frac{1 - \cos A}{2}}$$

$$= + \sqrt{\frac{1 - \frac{-4}{5}}{2}}$$

$$= \sqrt{\frac{1 + \frac{4}{5}}{2}} = \sqrt{\frac{5+4}{10}}$$

$$= \frac{\sqrt{9}}{\sqrt{10}} = \frac{3}{\sqrt{10}} = \frac{3\sqrt{10}}{10}$$



$$180^\circ < A < 270^\circ$$

$$90^\circ < \frac{A}{2} < 135^\circ$$

$\frac{A}{2}$ is in QII

$\sin > 0, \cos < 0$

Oct 9-10:56 AM

$$\cos \frac{A}{2} = \pm \sqrt{\frac{1 + \cos A}{2}} \quad \text{Since } \frac{A}{2} \text{ is in QII}$$

$$= -\sqrt{\frac{1 + \frac{-4}{5}}{2}} = -\sqrt{\frac{5-4}{10}} = -\frac{\sqrt{1}}{\sqrt{10}} = -\frac{\sqrt{10}}{10}$$

$\cos A = \frac{5}{13}$, A is in QIV

Find $\sin \frac{A}{2}$, $\cos \frac{A}{2}$.

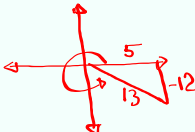
$$\sin \frac{A}{2} = +\sqrt{\frac{1 - \cos A}{2}}$$

$$= \sqrt{\frac{1 - \frac{5}{13}}{2}} = \sqrt{\frac{13-5}{13 \cdot 2}} = \sqrt{\frac{8}{26}} = \frac{2\sqrt{2}}{\sqrt{13}} = \frac{2\sqrt{26}}{13}$$

$$\cos \frac{A}{2} = -\sqrt{\frac{1 + \cos A}{2}} = -\sqrt{\frac{1 + \frac{5}{13}}{2}} = -\sqrt{\frac{13+5}{13 \cdot 2}}$$

$$= -\sqrt{\frac{9 \cdot 2}{13 \cdot 2}} = -\frac{3}{\sqrt{13}} = -\frac{3\sqrt{13}}{13}$$

$270^\circ < A < 360^\circ$
 $135^\circ < \frac{A}{2} < 180^\circ$
 $\frac{A}{2}$ is in QII
 $\sin > 0$
 $\cos < 0$



Oct 9-11:01 AM

$$\sin^2 A = \frac{1 - \cos 2A}{2} \Rightarrow \tan^2 A = \frac{\sin^2 A}{\cos^2 A} = \frac{1 - \cos 2A}{1 + \cos 2A}$$

$$\cos^2 A = \frac{1 + \cos 2A}{2}$$

$$\tan^2 A = \frac{1 - \cos 2A}{1 + \cos 2A}$$

$$\sin \frac{A}{2} = \pm \sqrt{\frac{1 - \cos A}{2}}$$

$$\cos \frac{A}{2} = \pm \sqrt{\frac{1 + \cos A}{2}}$$

$$\tan \frac{A}{2} = \pm \frac{\sin \frac{A}{2}}{\cos \frac{A}{2}}$$

$$\tan \frac{A}{2} = \frac{1 - \cos A}{\sin A} = \frac{\sin A}{1 + \cos A}$$

Determine + or - based on quadrant of $\frac{A}{2}$

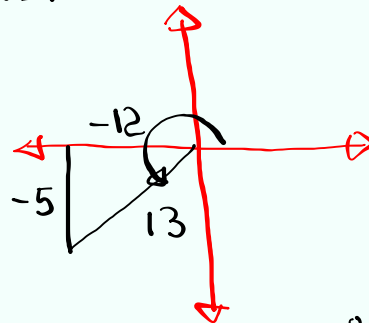
Oct 9-11:11 AM

$$\tan A = \frac{5}{12}, \quad A \text{ is in QIII}$$

Find $\tan \frac{A}{2}$

$$\tan \frac{A}{2} = \frac{1 - \cos A}{\sin A}$$

$$= \frac{1 - \frac{-12}{13}}{\frac{-5}{13}} = \frac{13 + 12}{-5} = \frac{25}{-5} = \boxed{-5}$$



$$180^\circ < A < 270^\circ$$

$$90^\circ < \frac{A}{2} < 135^\circ$$

QII

$\tan < 0$

Oct 9-11:16 AM

Find exact value of $\sin 22.5^\circ \rightarrow \text{QI}$

$$22.5^\circ = \frac{45^\circ}{2}$$

$$\sin 22.5^\circ = \sin \frac{45^\circ}{2} = \pm \sqrt{\frac{1 - \cos 45^\circ}{2}}$$

$$= + \sqrt{\frac{1 - \frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{2 - \sqrt{2}}{4}}$$

$$= \boxed{\frac{\sqrt{2 - \sqrt{2}}}{2}}$$

Oct 9-11:21 AM

Find $\cos \frac{x}{2}$ if $\sec x = \frac{3}{2}$, $270^\circ < x < 360^\circ$

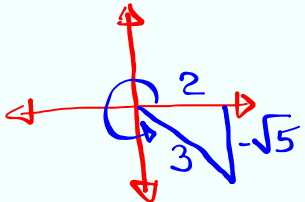
$$\cos \frac{x}{2} = -\sqrt{\frac{1 + \cos x}{2}}$$

$$= -\sqrt{\frac{1 + \frac{2}{3}}{2}}$$

$$= -\sqrt{\frac{3 + 2}{6}} = -\frac{\sqrt{5}}{\sqrt{6}} = -\frac{\sqrt{5}\sqrt{6}}{\sqrt{36}} = \boxed{-\frac{\sqrt{30}}{6}}$$

$\cos x = \frac{2}{3}$

$135^\circ < \frac{x}{2} < 180^\circ$
Q II
 $\cos < 0$



$2^2 + (-3)^2 = 3^2$
 $4 + 9 = 9$
 $9 = 9 \checkmark$

Oct 9-11:24 AM

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$\sin(A + B) + \sin(A - B) = 2 \sin A \cos B$$

Divide by 2

$$\boxed{\sin A \cos B = \frac{1}{2} [\sin(A + B) + \sin(A - B)]}$$

Product - to - Sum

Oct 9-11:35 AM